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Title: On Tallini's curve of degree $q+2$

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Abstract: Let $\mathcal{X}$ be an irreducible (possibly singular) plane curve defined over a finite field $\mathbb{F}_q$ such that $\mathcal{X}$ contains all points of $PG(2,q)$. By a result of G. Tallini dating back to 1960, $\deg \mathcal{X} \geq q+2$ and if equality holds then $\mathcal{X}$ is nonsingular and has a homogeneous equation $(aX_0+bX_1+cX_2)\varphi_{01} - X_0\varphi_{02} + X_2\varphi_{12} = 0$ where $\varphi_{ij} = X_i^q X_j - X_i X_j^q$ and a, b, c are elements in $\mathbb{F}_q$ such that the cubic polynomial $X^3 - cX^2 - aX - b$ is irreducible over $\mathbb{F}_q$. Tallini also showed that a Singer group Σ of $PG(2,q)$ is a (cyclic) automorphism group of $\mathcal{X}$ of order q^2+q+1. In 2003, Cossidente and Siciliano proved that all these curves of degree $q+2$ are projectively equivalent to the curve $\mathcal{X}_q$ of equation $X_1X_2^{q+1}+X_1^{q+1}X_0+X_2X_0^{q+1} = 0$, whenever the projective equivalence is considered over the cubic extension $\mathbb{F}_{q^3}$ of $\mathbb{F}_q$. Our contribution is to compute the Weierstrass semigroup of $\mathcal{X}_q$ at $Y_\infty = (0:0:1)$, and prove that $\mathcal{X}_q$ is a general curve, that is, its genus coincides with its Hasse-Witt invariant. We also consider quotient curves of $\mathcal{X}_q$ with respect to some subgroups of Σ. If $q = p^2$, for the subgroup G of Σ of order $p^2 - p + 1$ we prove that the quotient curve $\mathcal{X}/G$ is isomorphic to the Tallini curve of degree $p+2$ containing all points of the Baer-subplane $PG(2,p)$ of $PG(2,q)$.