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Title: On the a-number of Hurwitz and Fermat type curves

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Abstract: For an (algebraic, projective, absolutely irreducible) curve $\mathcal{X}$ defined over an algebraically closed field K of positive characteristic p, let $H^0(\mathcal{X}, \Omega_1)$ be the $g(\mathcal{X})$-dimensional vector space of holomorphic differentials on $\mathcal{X}$. The *Cartier operator* is the (unique) $1/p$-linear operator from $H^0(\mathcal{X}, \Omega_1)$ onto itself such that $C(\omega) = 0$ if and only if ω is exact and $C(\omega) = \omega$ if and only if ω is logarithmic. The *a-number* of $\mathcal{X}$ is the dimension $a(\mathcal{X})$ of the kernel of C, that is, the dimension of the space of holomorphic exact differentials. It is known that $a \leq g - \gamma$, where g is the genus and γ is the p-rank of $\mathcal{X}$. This suggests that the a-number may have deeper consequences on the structure of the Jacobian $\mathcal{J}(\mathcal{X})$ of $\mathcal{X}$. The explicit computation of a-numbers appears to be a challenging task. We present some new results on the a-numbers of two families of curves, namely the so-called Fermat type and Hurwitz type curves. A *Fermat curve* is the nonsingular plane curve $\mathcal{F}_n : X^n + Y^n + 1 = 0$ defined over the finite field $\mathbb{F}_p$ with $p \nmid n$, while a *Hurwitz curve* is the nonsingular plane curve $\mathcal{H}_n : X^nY + Y^n + X$ defined over $\mathbb{F}_p$ with $p \nmid n^2 - n + 1$. Such curves have been the subject of numerous investigations, especially in the case where their number of $\mathbb{F}_q$-rational points attains the famous Hasse-Weil bound on some extensions $\mathbb{F}_q$ of $\mathbb{F}_p$. Our main contribution is to give a necessary and sufficient conditions for a differential on $\mathcal{F}_n$ or $\mathcal{H}_n$ to be exact and compute explicitly the value of $a(\mathcal{F}_n)$ and $a(\mathcal{H}_n)$ for infinite values of n. As a byproduct, we recover the a-number of the Hermitian curve, which was originally found by Gross.