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Title: Large Automorphism Groups and p-rank of Curves

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Abstract: Let $\mathcal{X}$ be an (algebraic, projective, absolutely irreducible) curve defined over an algebraically closed field K of positive characteristic p. Let $\mathrm{Aut}(\mathcal{X})$ be the group of all automorphisms of $\mathcal{X}$ fixing K element-wise. By a classical result, $\mathrm{Aut}(\mathcal{X})$ is finite if the genus $\mathfrak{g}$ of $\mathcal{X}$ is at least two. Furthermore, if $\mathfrak{g} \geq 2$ and G is a subgroup of $\mathrm{Aut}(\mathcal{X})$ such that $p \nmid |G|$ then $|G| \leq 84(\mathfrak{g}-1)$.

By a result of Stichtenoth, $|\mathrm{Aut}(\mathcal{X})| < 16\mathfrak{g}^4$ with just one exception, namely the Hermitian curve of genus $\frac{1}{2}q(q-1)$. An improvement due to Henn states that $|\mathrm{Aut}(\mathcal{X})| > 8\mathfrak{g}^3$ only occurs for four curves. Each of these curves has zero p-rank.

We are interested in the study of curves with $|\mathrm{Aut}(\mathcal{X})| > c\mathfrak{g}^2$ for a constant $c > 42$ independent of $\mathfrak{g}$. Under such hypothesis, the quotient curve $\bar{\mathcal{X}} = \mathcal{X}/\mathrm{Aut}(\mathcal{X})$ is rational, and the cover $\mathcal{X}|\bar{\mathcal{X}}$ is non-tamely ramified at either one or two points of $\bar{\mathcal{X}}$. Our contribution is to prove that if this number is two then $\mathcal{X}$ has zero p-rank. Here, the p-rank of $\mathcal{X}$ (also called the Hasse-Witt invariant) is the integer γ so that the Jacobian of $\mathcal{X}$ has p^γ points of order p. It is known that $0 \leq \gamma \leq \mathfrak{g}$.