First Joint Meeting Brazil Italy of Mathematics Special Session 24: Geometric Variational and Evolution Problems

Organizers: Stefano Nardulli (UFRJ-Rio de Janeiro), Giandomenico Orlandi (Univr-Verona)

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Title: Sharp Riemmanian entropy inequalities

Authors: Marcos Montenegro

Abstract: We deals with Riemannian counterparts of the famous Gross's entropy inequality. We show how sharp entropy inequalities can be established as a limiting case of sharp Nash inequalities. Our method consists in controlling two optimal Nash constants and relies mainly on a careful analysis of concentration of minimizers to related energy functionals. This is joint work with Jurandir Ceccon.

Title: Constant nonlocal mean curvatures hypersurfaces

Authors: Mouhammed Moustapha Fall

Abstract: While Constant Mean Curvature (CMC) hypersurfaces are equilibrium hypersurfaces for the area functional, Constant Nonlocal Mean Curvatures (CNMC) appears when looking at equilibrium hypersurfaces for the nonlocal (or fractional) area functional. We will describe newly discovered CNMC hypersurfaces: those which have analogues in the theory of CMC (e.g. Delauney hypersurfaces) and those which do not have. These are joint works with: Xavier Cabré, Joan Sol-Morales and Tobias Weth.

Title: Isoperimetric Problems with Density

Authors: Frank Morgan

Abstract: Since their appearance in Perelman's proof of the Poincar Conjecture, there has been a huge surge of interest in spaces with density and the isoperimetric problem. We describe some recent results and open problems.

Title: Weighted tv minimization and applications to vortex density models

Authors: Giandomenico Orlandi

Abstract: Motivated by models arising from mathematical descriptions of Bose-Einstein condensation, we consider total variation minimization problems in which the total variation is weighted by a function that may degenerate near the domain boundary,

and the fidelity term contains a weight that may be both degenerate and singular. We develop a general theory for a class of such problems, with special attention to the examples arising from physical models. Joint with P. Athavale, R. Jerrard, M. Novaga.

Title: Effective energy of nearly-parallel Ginzburg-Landau vortex filaments

Authors: Robert Jerrard

Abstract: Starting from the Ginzburg-Landau model in cylindrical 3d domains, we derive an effective free energy functional for nearly-parallel vortex filaments. As a consequence, we establish the existence of solutions of the Ginzburg- Landau equations in certain scaling regimes, possessing a collection of vortex filaments minimizing this effective energy. This is joint work with Andres Contreras.

Title: Finite volume flows, Lefschetz Duality and Chern-Gauss-Bonnet

Authors: Daniel Cibotaru

Abstract: We will describe an extension of the work of Harvey and Lawson on finite volume flows to manifolds with boundary. We show that using an appropriate formalism with currents, Chern-Gauss-Bonnet on manifolds with boundary can be seen as a manifestation of Lefschetz Duality. Other applications of this formalism such as the construction of new, geometric Thom forms in both the even and the odd rank cases will also be presented.

Title: Topological line defects in nematic liquid crystals

Authors: Giacomo Canevari

Abstract: Nematic liquid crystals are composed by rod-shaped molecules with long-range orientation order. These materials exhibit topological defects, that is, discontinuities in the orientation parameter which depend on the topological behaviour of the configuration around them. A popular variational theory for nematic liquid crystals is the Landau-de Gennes theory, which shares common features with the Ginzburg-Landau functional for superconductivity and the harmonic maps problem. In particular, topological defects are associated with the homotopy groups of a manifold which, in the case under consideration, is a real projective plane. In this talk, we consider minimisers in a 3D-domain and take the limit as the material-dependent elastic constant tends to zero. We prove convergence to a map which has both point and line defects, and we describe the structure of line defects.

Title: Generalized solution in Variational problems

Authors: Vieri Benci

Abstract: In many circumstances, the notion of function is not su cient to the needs of a theory and it is necessary to extend it. We can recall, for example, the heuristic use of symbolic methods, called operational calculus developed by Oliver Heaviside in his book Electromagnetic Theory of 1899. The theory of Dirac and the theory of weak derivatives where unified by Schwartz in the beautiful theory of distributions, also thanks to the previous work of Leray and Sobolev. Among people working in partial differential equations, the theory of Schwartz has been accepted as definitive (at least until now), but other notions of generalized functions have been introduced by Colombeau and Mikio Sato. This conference deals with a new kind of generalized functions, called ultrafunctions, which have been recently introduced. The peculiarity of ultrafunctions is that they are based on the Non-Archimedean Mathematics (NAM). NAM is mathematics on fields which contain infinite and infinitesimal numbers (Non-Archimedan fields). The ultrafunctions have been

introduced to provide generalized solutions to equations which do not have any solutions not even among the distributions. In this talk, we will present some variational problems which do not have distribution solutions, but which have reasonable candidate solutions which can be interpreted as solutions in the framework of ultrafunctions.

Title: On stable CMC hypersurfaces with free-boundary in a Euclidean Ball

Authors: Ezequiel Barbosa

Abstract: In this talk, we will discuss how to prove that if B is a ball in a Euclidean space with dimension n, $n \geq 3$, then a stable CMC hypersurface Σ with free boundary in B satisfies

$$nA \leq L \leq nA\left(\frac{1+\sqrt{1+4(n+1)H^2}}{2}\right),$$

where L, A and H denote the length of $\partial\Sigma$, the area of Σ and the mean curvature of Σ, respectively. Consequently, if the boundary $\partial\Sigma$ is embedded then Σ must be totally geodesic or starshaped with respect to the center of the ball. This result is an improvement of a theorem proved by A. Ros and E. Vergasta In particular, if $n = 3$, the only stable CMC surfaces with free boundary in B are the totally geodesic disks or the spherical caps. This last result was proved very recently by I. Nunes using an extended stability result and a modified Hersch type balancing argument to get a better control on the genus. We don't use that modified Hersch type argument. However, we use a Nunes type Stability Lemma and a crucial result due to A. Ros and E. Vergasta.