FIRST JOINT MEETING BRAZIL–ITALY IN MATHEMATICS

SPECIAL SESSION #12: (NON)LOCAL MODELS AND APPLICATIONS

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Besides analysis of classical elliptic problems, the topics of the session include free boundary problems, calculus of variations and recent trends on local and nonlocal equations. In particular, in the last years a great interest has been devoted to the study of fractional and nonlocal operators of elliptic type. This interest is motivated both by the pure mathematical research and in view of concrete applications, since these operators arise in a quite natural way in many different contexts, such as, among the others, the thin obstacle problem, optimization, finance, phase transitions, stratified materials, anomalous diffusion, crystal dislocation, soft thin films, semipermeable membranes, flame propagation, conservation laws, ultra-relativistic limits of quantum mechanics, quasi-geostrophic flows, multiple scattering, minimal surfaces, materials science and water waves. The aim of this Special Session is to introduce recent progress and future applications in the field of local and nonlocal problems, inviting experts to discuss and exchange their ideas.

In what follows we recall the list of confirmed speakers:

- Bernhard Ruf (Università degli Studi di Milano)
- Claudianor Oliveira Alves (Universidade Federal de Campina Grande)
- Gaetano Siciliano (Universidade de São Paulo)
- João Marcos Bezerra do Ó (Universidade Federal da Paraiba)
- Liliane de Almeida Maia (Universidade de Brasília)
- María Medina (Pontificia Universidad Católica de Chile)
- Olìmpio Hiroshi Miyagaki (Universidade Federal de Juiz de Fora)
- Sandra Imaculada Moreira Neto (Universidade Estadual do Maranhão)

The duration of each seminar of our session will be of 25 minutes (plus 5 minutes for questions).

Abstracts of Seminars

Title: On a heat equation with exponential nonlinearity in $\mathbb{R}^2$

Authors: Bernhard Ruf

Abstract: We consider a semilinear heat equation with exponential nonlinearities in $\mathbb{R}^2$.

In $\mathbb{R}^N, N \geq 3$, critical growth is polinomial, and has been studied by several authors: existence and non-existence results were obtained for singular initial data in suitable L^p-spaces by F. Weissler and H. Brezis–T. Cazenave; furthermore, non-uniqueness results were obtained by W.-M. Ni–P. Sacks and E. Terraneo for certain singular initial data.

In $N = 2$ critical growth is given by nonlinearities of exponential type. With prove that similar phenomena occur for suitable exponential nonlinearities and singular initial data in certain Orlicz spaces.

Title: On a Class of Intermediate Local-Nonlocal Elliptic Problems

Authors: Claudianor O. Alves, Francisco Julio S. A. Corrêa and Michel Chipot

Abstract: In this talk, we will show the the existence of solutions for a class of local-nonlocal boundary value problems of the following type

$$(IP) \qquad -div\left[a\left(\fint_{\Omega(x,r)} u(y)dy\right)\nabla u\right] = f(x, u, \nabla u) \text{ in } \Omega, \quad u \in H_0^1(\Omega)$$

where Ω is a smooth bounded domain of $\mathbb{R}^N$, $a : \mathbb{R} \to \mathbb{R}$ is a continuous function, $f : \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$ is a given function, $r > 0$ is a fixed number, $\Omega(x, r) = \Omega \cap B(x, r)$, where $B(x, r) = \left\{y \in \mathbb{R}^N; |y - x| < r\right\}$. Here $|\cdot|$ is the Euclidian norm, $\fint_{\Omega(x,r)} u(y)dy = \frac{1}{|\Omega(x, r)|}\int_{\Omega(x,r)} u(y)dy$ and $|X|$ denotes the Lebesgue measure of a measurable set $X \subset \mathbb{R}^N$.

Title: Semiclassical states for a fractional Schrödinger-Poisson system

Authors: Edwin G. Murcia, Gaetano Siciliano

Abstract: We consider the following doubly singularly perturbed fractional Schrödinger-Poisson system in $\mathbb{R}^N$:

$$\begin{cases} \varepsilon^{2s}(-\Delta)^s w + V(x)w + \psi w = f(w) \\ \varepsilon^{\theta}(-\Delta)^{\alpha/2}\psi = w^2, \end{cases}$$

in the unknown w, ψ. Here ε is a positive parameter, V an external potential and f a nonlinearity. Under suitable assumptions on $V, f, s, \alpha, \theta, N$ we show by variational methods that, for sufficiently small ε, the number of positive solutions of the system is estimated below by the Ljusternick-Schnirelmann category of the set of minima of the potential V.

Title: Concentration-compactness principle for nonlocal scalar field equations with critical growth

Authors: João Marcos do Ó & Diego Ferraz

Abstract: We study a concentration-compactness principle for homogeneous fractional Sobolev space $\mathcal{D}^{s,2}(\mathbb{R}^N)$ for $0 < s < N/2$. As an application we establish Palais-Smale compactness for the Lagrangian associated to the fractional scalar field equation $(-\Delta)^s u = f(x,u)$ for $0 < s < 1$. Moreover, using an analytic framework based on $\mathcal{D}^{s,2}(\mathbb{R}^N)$, we obtain existence results for a wide class of nonlinearities in the critical growth range.

Title: The effect of the Hardy potential in some Calderón-Zygmund properties for the fractional Laplacian.

Authors: María Medina

Abstract: In this talk we will consider the nonlocal problem

$$\begin{cases} (-\Delta)^s u - \lambda \dfrac{u}{|x|^{2s}} = f(x) & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \\ u > 0 & \text{in } \Omega, \end{cases}$$

where $s \in (0,1)$, Ω is a smooth bounded domain containing the origin and $N > 2s$. We will study its solvability, and the summability of the corresponding solution, according to the integrability of f, in the spirit of the classical results of Stampacchia. In particular, we will try to find the optimal conditions on f and λ in order to obtain a solution, seeing that the techniques applied in (2), where the authors study this problem in the case $s = 1$, do not provide optimality in this setting.

This work can be found in (1).

References:

(1) B. Abdellaoui, M. Medina, A. Primo, I. Peral, *The effect of the Hardy potential in some Caldern-Zygmund properties for the fractional Laplacian*. Journal of Differential Equations, 260 (2016), pp. 8160-8206. DOI: 10.1016/j.jde.2016.02.016.

(2) L. Boccardo, L. Orsina, I. Peral, *A remark on existence and optimal summability of solutions of elliptic problems involving Hardy potential*. Discrete and continuous dynamical systems, Volume 16, Number 3 (2006), pp. 513-523.

Title: On fractional p-Laplacian problems with weight

Authors: Raquel Lehrer, Liliane A. Maia and Marco Squassina

Abstract: We will present recent results on the existence of nonnegative solutions for a nonlinear problem involving the fractional p-Laplacian operator. The problem is set on a unbounded domain, and compactness issues have to be handled. This work is in collaboration with Raquel Lehrer from UNIOESTE, Brazil, and Marco Squassina from Università degli Studi di Verona, Italy.

Title: Nonlinear fractional elliptic equation with saddle-like potentical in $\mathbb{R}^N$

Authors: Claudianor O. Alves (UFCG-BRASIL) and Olímpio H. Miyagaki (UFJF-BRASIL)

Abstract: In this paper, we study the existence of positive solution for the following class of fractional elliptic equation

$$\epsilon^{2s}(-\Delta)^s u + V(z)u = \lambda |u|^{q-2}u + |u|^{2_s^*-2}u \text{ in } \mathbb{R}^N,$$

where $\epsilon, \lambda > 0$ are positive parameters, $q \in (2, 2_s^*)$, $2_s^* = \frac{2N}{N-2s}$, $N > 2s$, $s \in (0,1)$, $(-\Delta)^s u$ is the fractional laplacian, and V is a saddle-like potential. The result is proved by using minimizing method constrained to the Nehari manifold. A special minimax level is obtained by using an argument made by Benci and Cerami.

Remarks: Research of C. O. Alves partially supported by CNPq/Brazil Proc. 304036/2013-7 and INCTMAT/CNPq/Brazil.

O.H.M. was partially supported by INCTMAT/CNPq/Brazil, CNPq/Brazil Proc . 304014/2014-9 and CAPES/Brazil Proc. 2531/14-3.

Title: Multiplicity of nonnegative solutions for quasilinear Schrödinger equations

Authors: Olímpio H. Miyagaki, Sandra Im. Moreira and Patrizia Pucci

Abstract: We establish the existence and multiplicity of nonnegative weak solutions for quasilinear Schrödinger equations involving nonlinearities with possibly supercritical growth at infinity and indefinite sign. By changing the variables, the quasilinear equations are reduced to a semilinear form and variational methods are then applied in order to obtain the main results.