

First Joint Meeting Brazil Italy of Mathematics
Special Session: Variational and geometric
methods

Rio de Janeiro, August 29 - September 02, 2016

Title: Existence and concentration phenomena for a GKP equation in \mathbb{R}^2 .

Authors: Claudianor O. Alves and Olimpio H. Miyagaki.

Abstract: In this talk, we intend to show some results concerning the existence, concentration phenomena and regularity of solutions for a *Generalized Kadomtsev Petviashvili* equation - GKP in \mathbb{R}^2 .

Title: A Faber-Krahn inequality for solutions of Schrödinger's equation on Riemannian manifolds

Authors: Emerson Abreu and Ezequiel Barbosa

Abstract: We consider a bounded open set with smooth boundary $\Omega \subset M$ in a Riemannian manifold (M, g) , and suppose that there exists $u \in C(\Omega)$ solving the problem

$$-\Delta u = V(x)u, \text{ in } \Omega,$$

in the distributional sense, with $V \in L^\infty(\Omega)$, where $u \equiv 0$ on $\partial\Omega$. We prove a sharp inequality involving $\|V\|_\infty$ and the first eigenvalue of the Laplacian for special domains, which generalizes the well known Faber-Krahn inequality.

Title: Henon equation with Neumann boundary condition

Authors: Jaeyoung Byeon

Abstract: We consider the Henon equation on a bounded domain with the homogeneous Neumann boundary condition. One of basic concerns on the equation is to understand asymptotic behavior of least energy solutions when $\alpha \rightarrow \infty$ for the nonautonomous term $|x|^\alpha$. In this talk, I would like to explain that as $\alpha \rightarrow \infty$, many new different types of asymptotic behavior of least energy solutions arise depending on a geometry of the domain and the growth rate of a nonlinear term.

Title: Ruf's cubic perturbation of the Laplacian yields a global cusp

Authors: Marta Calanchi

Abstract: Following the paper of Ruf [1], let $F(u) = -\Delta u - f(u)$, where f is a specific cubic nonlinearity. Ruf was one step away from proving the existence of global coordinates both in domain and counterdomain (Holder spaces with

Neumann boundary conditions) converting F into a map $(x, y, v) \mapsto (x, y^3 - xy, v)$ of $\mathbb{R} \times \mathbb{R} \times X$ into itself. We provide the missing step. Joint work with Nicolau Saldanha and Carlos Tomei.

[1] B. Ruf, Singularity theory and the geometry of a nonlinear elliptic equation, Ann. Scuola Norm. Sup. Pisa Cl. Sc., (2), 17 (1990), 1-33.

Title: $\mathcal{D}^{1,2}(\mathbb{R}^N)$ versus $C(\mathbb{R}^N)$ Local Minimizer and a Hopf-Type Maximum Principle

Authors: C. Siegfried, David G. Costa and H. Tehrani

Abstract: We consider functionals of the form $\Phi(u) = \frac{1}{2} \int_{\mathbb{R}^N} |\nabla u|^2 - \int_{\mathbb{R}^N} b(x)G(u)$ on $\mathcal{D}^{1,2}(\mathbb{R}^N)$, $N \geq 3$, whose critical points are the weak solutions of a corresponding elliptic equation in the whole \mathbb{R}^N . We present a Brezis-Nirenberg type result and a Hopf-type maximum principle in the context of the space $\mathcal{D}^{1,2}(\mathbb{R}^N)$. More precisely, we prove that a local minimizer of Φ in the topology of the subspace V must be a local minimizer of Φ in the $\mathcal{D}^{1,2}(\mathbb{R}^N)$ -topology, where V is given by $V := \{v \in \mathcal{D}^{1,2}(\mathbb{R}^N) : v \in C(\mathbb{R}^N) \text{ with } \sup_{x \in \mathbb{R}^N} (1 + |x|^{N-2})|v(x)| < \infty\}$

Title: Existence of a ground state solution for a problem involving 1-Laplacian operator.

Authors: Giovany M. Figueiredo and Marcos T. O. Pimenta

Abstract: We investigate the existence of ground states for functionals that is not C^1 class. Roughly speaking, we show that the Nehari manifold method requires no regularity of the functional. As an application, we prove the existence of a ground state solution for a problem involving the 1-Laplacian operator.

Title: On Yamabe type problems on Riemannian manifolds with boundary

Authors: Marco Ghimenti

Abstract: Let (M, g) be a n -dimensional compact Riemannian manifold with boundary. We consider the following problem in the slightly supercritical and slightly subcritical case

$$\begin{cases} -\Delta_g u + au = 0 & \text{on } M \\ \partial_\nu u + \frac{n-2}{2}bu = (n-2)u^{\frac{n}{n-2} \pm \varepsilon} & \text{on } \partial M \end{cases}$$

This problem represents the analogous of Yamabe problem when the manifold has a non empty boundary.

We build solutions which blow-up at a point of the boundary as the parameter ε goes to zero. The blowing-up behavior is ruled by the function $b - H$, where H is the boundary mean curvature. The proof of the result relies on a Ljapunov-Schmidt procedure.

Title: A nonvariational system involving the critical Sobolev exponent

Authors: Massimo Grossi

Abstract: We consider the non-variational system

$$\begin{cases} -\Delta u_i = \sum_{j=1}^k a_{ij} u_j^{\frac{N+2}{N-2}} & \text{in } \mathbb{R}^N, \\ u_i > 0 & \text{in } \mathbb{R}^N, \\ u_i \in D^{1,2}(\mathbb{R}^N). \end{cases} \quad (1)$$

and we give some sufficient conditions on the matrix $(a_{ij})_{i,j=1,\dots,k}$ which ensure the existence of solution bifurcating from the bubble of the critical Sobolev equation. This is a joint paper with F. Gladiali and C. Troestler.

Title: Positive Solutions for Asymptotically Linear Problems in Exterior Domains

Authors: Liliane A. Maia and Benedetta Pellacci

Abstract: We will present some recent results on the existence of a positive solution for the following class of elliptic problems

$$-\Delta u + \lambda u = f(u), \quad \text{in } \Omega, \quad u \in H_0^1(\Omega),$$

where Ω is an unbounded domain in \mathbb{R}^N not necessarily symmetric, $N \geq 3$, with smooth boundary $\partial\Omega \neq \emptyset$ bounded, and such that $\mathbb{R}^N \setminus \Omega$ is bounded. The non-linearity f is super-linear at zero and asymptotically linear at infinity. This result is established via a linking argument on the Nehari manifold and by means of a barycenter function. This is a work in collaboration with Benedetta Pellacci from Università degli Studi di Napoli *Parthenope*, Italy.

Title: Quasilinear elliptic equations with critical potentials

Authors: Enzo Mitidieri

Abstract

Variants of Kato's inequality are proved for general quasilinear elliptic operators L . As an outcome we show that, dealing with Liouville theorems for coercive equations of the type

$$Lu = f(x, u, \nabla_L u) \quad \text{on } \Omega \subset \mathbb{R}^N,$$

where f is such that $f(x, t, \xi) t \geq 0$, the assumption that the possible solutions are nonnegative involves no loss of generality. Related consequences such as comparison principles, *a priori* bounds on solutions and implication to Schrödinger type equations are also presented. An underlying structure throughout this work is the framework of Carnot groups.

Title: A non-compactness result on the fractional Yamabe problem in large dimensions

Authors: Monica Musso

Abstract: Let (X^{n+1}, g^+) be an $(n+1)$ -dimensional asymptotically hyperbolic manifold with a conformal infinity (M^n, h) . The fractional Yamabe problem addresses to solve

$$P^\gamma[g^+, h](u) = cu^{\frac{n+2\gamma}{n-2\gamma}}, \quad u > 0 \quad \text{on } M$$

where $c \in \mathbb{R}$ and $P^\gamma[g^+, h]$ is the fractional conformal Laplacian whose principal symbol is $(-\Delta)^\gamma$. In this paper, we construct a metric on the half space $X = \mathbb{R}_+^{n+1}$, which is conformally equivalent to the unit ball, for which the solution set of the fractional Yamabe equation is non-compact provided that $n \geq 24$ for $\gamma \in (0, \gamma^*)$ and $n \geq 25$ for $\gamma \in [\gamma^*, 1)$ where $\gamma^* \in (0, 1)$ is a certain transition exponent. The value of γ^* turns out to be approximately 0.940197. This is a joint work with S. Kim and J. Wei.

Title: Morse index of sign changing solutions of semilinear elliptic problems

Authors: Filomena Pacella

Abstract: We will present recent results about the computation of the Morse index of radial sign changing solutions in the ball of Lane-Emden problems. The method used relies in analyzing some limit weighted eigenvalue problems in the whole space which naturally arises when studying the linearized operator for asymptotic values of the exponent of the nonlinearity. The results have been obtained in collaboration with F.De Marchis and I.Ianni.

Title: Morse index of sign changing solutions of semilinear elliptic problems

Authors: Benedetta Pellacci

Abstract: Optimization of the positive principal eigenvalue for fractional Neumann problems We will study Neumann boundary value problems under the action of fractional diffusion. This kind of model are particularly suitable to study populations dynamic when long jumps, the so-called Lévy flights, are admitted in order to search prey. We will focus on the study of the optimization of the positive principal eigenvalue in dependence on the indefinite potential, on the motility function and on the fractional exponent. This analysis is related to the optimization of the survival threshold in populations dynamic.

Title: Blowing-up solutions for Yamabe-type problems

Authors: Angela Pistoia

Abstract: The Yamabe equation is one of the most natural and well-studied second-order semilinear elliptic equations arising in geometric variational problems. The issue of the compactness of the set of solutions of the geometric Yamabe equation has been recently studied and it is strictly related to the existence of solutions blowing-up at one or more points in the manifold. In this lecture, I will review these results and present more recent works on the Yamabe problem, where solutions blowing-up at multiple (clustering and towering) points have been found.

Title: On the regularity and asymptotic behavior of the solutions to some parabolic PDE

Authors: Maria Michaela Porzio

Abstract: It is well known that the heat equation exhibits a very strong regularization phenomenon: the solutions become "immediately bounded" also in presence of only summable initial data.

Indeed, this "strong regularizing effect" is not a peculiarity of the heat equation since it appears also for a lot of other parabolic problems, also nonlinear,

degenerate or singular like degenerate p -Laplacian equation, the porous medium equation, etc. In the same time there are evolution problems for which this regularization does not appear like some singular p -Laplacian or fast diffusion equations. We show a new method to describe this phenomenon and to derive regularity estimates. Moreover, we investigate what happens when this regularizing effect does not appear and which is the solutions' behavior in all these different cases.

Title: Some classical inequalities revisited in the fractional Laplacian framework

Authors: Olivaine S. de Queiroz

Abstract: We are interested in the study of some classical inequalities such as Sobolev-Trudinger-Moser and also the Faber-Khran in the fractional Laplacian framework. We apply our results in the study of some free boundary problems and also in some nonlinear PDE's from Conformal Geometry.