First Joint Meeting Brazil Italy of Mathematics
Special Session: Geometric Topology and Dynamics*

Rio de Janeiro, August 29 - September 02, 2016

Title: Involutions on closed Sol 3-manifolds and the Borsuk-Ulam Theorem for maps into $\mathbb{R}^n$

Authors: Alexandre Paiva Barreto (speaker), D. L. Gonçalves, D. Vendrúscolo

Abstract: In this talk we classify the free involutions of a Sapphire Sol 3-manifold. We determine also whether a triple $(M, \tau; \mathbb{R}^n)$, where M is a Sol 3-manifold and τ is a free involution on M, has the Borsuk-Ulam Property or not.

Title: Grids and branched coverings of the sphere

Authors: Natalia A. Viana Bedoya

Abstract: In this work we study $m \times n$ grids of nodes and edges realizing branched coverings of the sphere. We will introduce this relation and their properties.

Title: Counting isospectral manifolds

Authors: Mikhail Belolipetsky (speaker), Benjamin Linowitz

Abstract: I will talk about a recent joint work with Benjamin Linowitz, in which we show that surprisingly many higher rank locally symmetric spaces are mutually isospectral.

Title: The Riemann-Hilbert mapping on $\mathfrak{sl}_2$-systems

Authors: G. Calsamiglia (speaker), B. Deroin, V. Heu and F. Loray

Abstract: The Riemann-Hilbert mapping on $\mathfrak{sl}_2$-systems associates, to any $\mathfrak{sl}_2$ connection on a trivial bundle $X \times \mathbb{C}^2$ over a genus $g \geq 2$ Riemann surface X, the class of its monodromy representation in the SL_2 character variety.

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We prove that the Riemann-Hilbert mapping is a local diffeomorphism around any point of genus $g = 2$ with irreducible monodromy.

Title: Dynamics, braids and 3-manifolds

Author: André de Carvalho

Abstract: In this talk we will discuss how *implication* organizes braid types of periodic orbits of Smale's horseshoe and relate this to the hyperbolic structures on the associated suspension 3-manifolds. We will also discuss taking limits in both worlds: of braid types and of hyperbolic 3-manifolds.

Let S be a surface, $f\colon S \to S$ a homeomorphism and $P \subset S$ an f-invariant subset. The *braid type* $\mathrm{bt}(f, P)$ is the isotopy class of f relative to P up to topological changes of coordinates. If S is the plane and P is a finite set, then $\mathrm{bt}(P, f)$ can be thought of as a geometric braid, hence the name. The mapping torus $\mathrm{mt}(f, P)$ is the suspension of the homeomorphism $f|_{S\setminus P}\colon S \setminus P \to S \setminus P$, i.e., the 3-manifold obtained taking the product $(S\setminus P)\times[0,1]$ and identifying $(x, 1)$ and $(f(x), 0)$ for all $x \in S\setminus P$. Thurston's classification of surface homeomorphisms up to isotopy allows us to define the partial order of *implication* among braid types. Understanding this partial order is central to the understanding of how dynamics builds up in families of surface diffeomorphisms, such as the Hénon family for example. Thurston's hyperbolization theorem for fibered 3-manifolds (often) finds hyperbolic structures on the mapping tori $\mathrm{mt}(f, P)$. The dynamical information encoded in braid types is strongly related to the associated mapping tori, but this relationship is very non-obvious. In particular, it makes sense to take limits of sequences of braid types and of hyperbolic 3-manifolds, but the relationship between the limiting objects is still rather mysterious. We will present examples of converging sequences in both cases and discuss how to try to reconcile the apparent inconsistencies.

Title: The theory of train tracks and the conjugacy problem for automorphisms of free groups

Authors: Stefano Francaviglia (speaker), Armando Martino

Abstract: In this talk we will discuss the topological approach to the study of outer automorphisms of free groups given by the study of free homotopy classes of maps between graphs. The moduli space of marked graphs with fundamental group F_n is know as Culler-Vogtmann Outer Space, and the group of outer automorphism of F_n acts by isometries on that space. We will show how the axis of $\phi \in Out(F_n)$ — that is to say the points of the outer space that are minimally displaced by ϕ — can be described in terms of train-track maps (introduced in this setting by Bestvina and Handel), and how train tracks are a powerful tool in studying automorphisms, with particular attention to the reducibility problem (solved by I. Kapovich) and the conjugacy problem in $Out(F_n)$ (solved by J. Loss for irreducible automorphisms).

Title: Ergodic actions and intergral approximations of Gromov's simplicial volume

Authors: R. Frigerio

Abstract: The simplicial volume is a homotopy invariant of closed manifolds defined by Gromov in 1982. For a manifold M, it is bounded from above by the minimal number of top-dimensional simplices in a triangulation of M, and roughly speaking it measures the minimal size of triangulations of M "with real coefficients". A long-standing conjecture by Gromov asserts that, for aspherical manifolds, the vanishing of the simplicial volume implies the vanishing of the Euler characteristic. In this talk I describe an approach to this conjecture that makes use of discrete approximations of the simplicial volume in towers of coverings, as well as of ergodic actions of the fundamental group of M on suitable probability spaces.

Title: Tunnel number of knots from a combinatorial perspective

Authors: Darlan Girão (speaker), J. Nogueira, A. Salgueiro

Abstract: In this talk we discuss a combinatorial approach for the problem of computing the tunnel number of knots and links in S^3. By considering minimal crossing diagrams, we study properties associated to *percolation* in graph theory and show how this gives estimates on tunnel number. We then show how this behaves in particular classes of links.

Title: Aspherical products which do not support Anosov diffeomorphisms

Authors: J.-F. Lafont (speaker), A. Gogolev

Abstract: A famous conjecture of Smale predicts that the only closed manifolds that support Anosov diffeomorphisms are infranilmanifolds. I will explain why the product of infranilmanifolds with certain aspherical closed manifolds do not support Anosov diffeomorphisms. As a special case, we obtain that products of a nilmanifold and negatively curved manifolds of dimension at least three do not support Anosov diffeomorphisms. This is joint work with Andrey Gogolev (Binghamton University).

Title: Hyperbolic four-manifolds

Authors: Bruno Martelli

Abstract: We survey some recent constructions of hyperbolic four-manifolds, focusing on some aspects that are familiar to three-dimensional topologists: cusp shapes, volumes,

Dehn filling. The main technique is the assembling of Coxeter hyperbolic four-dimensional polytopes.

Title: Persistent Homology and Barcode Fields
Authors: Washington Mio (speaker), Mao Li
Abstract: Persistent homology lets us construct informative summaries of the geometry and topology of data and probability measures via barcodes. We propose a local-to-global form of homology represented by continuous barcode fields that are stable with respect to small perturbations of a probability measure as measured by the Wasserstein distance. We also discuss applications to shape analysis.

Title: Surfaces with spherical metrics
Authors: Gabriele Mondello (speaker), Dmitri Panov
Abstract: McOwen and Troyanov proved existence and uniqueness of conformal metrics of constant non-positive curvature and conical singularities of prescribed angles on every compact Riemann surface with marked points, provided the obvious Gauss-Bonnet constraint is satisfied. Thus, the moduli space of such metrics can be essentially identified to the moduli space of Riemann surfaces with marked points.

Quite differently, in constant positive curvature no such existence and uniqueness hold in general. I will illustrate that there is a natural obstruction to the existence, which can be easily expressed in terms of the angles. Moreover, given angles $\underline{\vartheta}$ for which such obstruction vanishes, I will discuss some properties of the moduli space of surfaces of genus g with a metric of curvature 1 and conical points of angles $\underline{\theta}$.

Title: Reidemeister torsion, hyperbolic three-manifolds, and character varieties
Authors: Joan Porti
Abstract: In this talk I plan to overview recent results on the behavihor of Reidemeister torsions from two viewpoints: as topological invariants of closed hyperbolic three-manifolds, and also as functions on the variety of representations of cusped manifolds in $\mathrm{SL}_2(\mathbb{C})$. In addition, I consider torsions obtained after composing with a finite dimensional representation of $\mathrm{SL}_2(\mathbb{C})$.

Title: A Central Limit Theorem for acylindrically hyperbolic groups
Authors: Alessandro Sisto (speaker), Pierre Mathieu

Abstract: Acylindrically hyperbolic groups form a very large class of groups that includes non-elementary (relatively) hyperbolic groups, mapping class groups, $Out(F_n)$, many groups acting on CAT(0) spaces, etc. I will discuss the behaviour of random walks on such groups, in particular illustrating the fact that random paths tend to stay close to geodesics, and I will explain how this can be used to get a central limit theorem for the distance from the identity of the random walk.

Title: Tessellating moduli spaces of strictly convex projective structures

Authors: Stephan Tillmann (speaker) Sampson Wong, Robert Haraway

Abstract: Associating the Euclidean cell decomposition due to Cooper and Long to each point of the moduli space of framed strictly convex real projective structures of finite volume on a non-compact manifold gives this moduli space a natural decomposition. I will describe algorithms to compute these decompositions for surfaces and discuss some applications.

Title: On finite group actions on surfaces, finite graphs and 3-manifolds

Authors: Bruno P. Zimmermann (speaker), Chao Wang, Shicheng Wang, Yimu Zhang

Abstract: We report on two recent results on finite group actions of large order on surfaces and finite graphs embedded in S^3, and on closed 3-manifolds:

i) By a classical result of Hurwitz, the maximal possible order of a finite orientation-preserving group action on a closed orientable surface F_g of genus $g > 1$ is $84(g-1)$. In a program to visualize finite group actions of large orders on surfaces, in joint work with Chao Wang, Shicheng Wang and Yimu Zhang, we consider finite group actions on pairs (S^3, F_g) where F_g is a closed surface of genus g embedded in S^3. An upper bound for the order is $12(g-1)$ now, and we determine the maximal order of such an action for each $g > 1$; in fact, we classify all finite group actions on such pairs (S^3, F_g) of orders larger than $4(g-1)$. Then we give a similar classification for finite group actions on finite graphs of rank $g > 1$ embedded in S^3.

ii) If there remains time, we will discuss also joint work with M. Boileau, C. Franchi, M. Mecchia and L. Paoluzzi where we show that, given a closed 3-manifold M, there is a universal bound on the number of inequivalent knots in S^3 which have M as a cyclic branched covering. The most interesting and difficult case here is that of a hyperbolic 3-manifold M (in particular, with a finite group of isometries) where one has to rely heavily on methods from finite group theory, including the classification of the finite simple groups.