First Joint Meeting Brazil Italy of Mathematics
Special Session: Inverse Problems for PDEs

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Title: Lipschitz stability for the electrostatic inverse boundary value problem with piecewise linear conductivities

Authors: Eva Sincich

Abstract: We consider the inverse boundary value problem associated with the elliptic equation for an electric potential, where the objective is to recover the conductivity from partial data. We focus our attention on the stability of this inverse problem, in particular, when the conductivity is isotropic. We obtain a Lipschitz stability result if the conductivity is known to be piecewise linear on a given domain partition.

We let Ω be a bounded domain in $\mathbb{R}^n$, $n \geq 2$. In the absence of internal sources, the electric potential, u, satisfies the elliptic equation

$$\operatorname{div}(\gamma \nabla u) = 0 \qquad \text{in} \quad \Omega, \tag{1}$$

where the function γ signifies the *conductivity* in Ω; γ is a bounded measurable function satisfying the ellipticity condition,

$$0 < \lambda^{-1} \leq \gamma \leq \lambda, \qquad \text{almost everywhere in } \Omega, \tag{2}$$

for some positive $\lambda \in \mathbb{R}$. The inverse conductivity problem consists of finding γ when the so-called Dirichlet-to-Neumann (DtoN) map

$$\Lambda_\gamma : \ u|_{\partial\Omega} \in H^{\frac{1}{2}}(\partial\Omega) \longrightarrow \gamma \nabla u \cdot \nu|_{\partial\Omega} \in H^{-\frac{1}{2}}(\partial\Omega) \tag{3}$$

is given for any weak solution $u \in H^1(\Omega)$ to (1). Here, ν denotes the unit outward normal to $\partial\Omega$. If measurements can be taken on a portion Σ of $\partial\Omega$ only, then the relevant map is referred to as the local DtoN map.

Title: The inverse conductivity problem for anisotropic materials.

Authors: Romina Gaburro

Abstract: We consider the inverse problem of determining the conductivity of a body by taking measurements of voltage and current on its surface. This means, in mathematical terms, to solve an inverse boundary value problem where the unknown is the coefficient

matrix (the conductivity) of a partial differential equation (the conductivity equation), from the knowledge of the so-called Dirichlet-to-Neumann (DtoN) map (the data or measurements). Different materials display different electrical properties, so that a map of the conductivity can be used to investigate internal properties of the body under investigation and to obtain an image of its interior. This inverse problem has came to be known as Electrical Impedance Tomography (EIT). EIT has many important applications in fields such as geophysics, medicine and nondestructive testing of materials. Although is well known that this problem is severely ill-posed, quite a lot of progress has been made for the case when the body to be imaged is filled with isotropic material. In this talk we will focus our attention on the anisotropic case and will present some recent results of uniqueness and stability for certain type of anisotropic materials.

Title: On compact hypersurfaces with almost constant mean curvature

Authors: Giulio Ciraolo

Abstract: Alexandrovs theorem asserts that spheres are the only closed compact embedded hypersurfaces with constant mean curvature in the Euclidean space. In this talk we will discuss some quantitative versions of Alexandrovs theorem, i.e. we will consider a hypersurface with mean curvature close to a constant and quantitatively describe its proximity to a sphere or a collection of tangent spheres of equal radii in terms of the oscillation of the mean curvature.

We will also discuss these issues for the nonlocal mean curvature, showing a remarkable rigidity property of the nonlocal problem which prevents bubbling phenomena and proving the proximity to a single sphere.

Title: Inexact Newton combined with gradient methods in Banach spaces

Authors: Fábio Margotti

Abstract: Inexact Newton methods have proven to be a powerful class of iterative methods for solving nonlinear ill-posed problems in Hilbert spaces. In order to realize such a method, one must linearize the original equation around the current iterate and then apply a regularization technique to solve the resulting linear system. We propose the adaptation of some classical gradient-type regularization methods for solving the linear systems in a relatively general Banach space setting.

Title: A stability result for quantitative photoacoustic tomography

Authors: Michele Di Cristo

Abstract: We treat the stability issue for the three dimensional inverse imaging modality called Quantitative Photoacoustic Tomography. We provide universal choices of the illuminations which enable to recover, in a Hölder stable fashion, the diffusion and absorption coefficients from the interior pressure data. With such choices of illuminations we do not need the nondegeneracy conditions commonly used in previous studies, which are difficult to be verified a-priori.

Title: Inverse Problems for Parabolic Partial Differential Equations: An Application to Dupire's model

Authors: Jorge Zubelli

Abstract: Local volatility models are extensively used and well-recognized for hedging and pricing in financial markets. They are frequently used, for instance, in the evaluation

of exotic options so as to avoid arbitrage opportunities with respect to other instruments. From a mathematical viewpoint, the problem consists in estimating a variable diffusion coefficient (the so-called volatility) in a parabolic PDE from observed solutions of such equation.

The ill-posed character of local volatility surface calibration from market prices requires the use of regularization techniques either implicitly or explicitly. Such regularization techniques have been widely studied for a while and are still a topic of intense research. The subject falls naturally in the beautiful intersection of Statistics, Optimization, Numerical Analysis and PDEs.

In the final part of the talk we shall describe ongoing work on the use local volatility models in the context of commodity markets, in particular applied to energy and oil ones. This work is part of ongoing collaboration with V. Albani (Vienna), U. Ascher (Toronto), and Xu Yang (IMPA).

Title: TBA

Authors: Antonio Leitao

Abstract: TBA