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Title: A survey of recent results on the characterization of decay of solutions to dissipative equations

Authors: César J. Niche - UFRJ

Abstract: Solutions to many dissipative equations in Fluid Mechanics, like the Navier-Stokes, quasi-geostrophic and Navier-Stokes-Voigt equations, obey energy inequalities that imply that their L^2 or Sobolev norms decay in time. In the late 80s M.E. Schonbek developed the Fourier Splitting method, which has been widely used to establish decay rates for these and many other systems of equations.

The Fourier Splitting method is based on the idea that "long time behavior of solutions is determined by small frequencies and for it to provide uniform decay rates, it is necessary to restrict initial data to subsets of L^2 (say, $L^p \cap L^2, 1 \leq p < 2$, or data for which the linear part of the equations has certain decay). The question that then naturally arises is whether it is possible to establish decay for *any* initial data in L^2.

In this talk we will survey recent work in which the decay rates are characterized, for *any* initial data, for solutions to many families of dissipative equations. This description is based on the *decay character* $r^* = r^*(u_0)$ associated to the initial datum u_0. This number is, rougly speaking, the order of u_0 at the origin in frequency space and can be used to give explicit upper (and sometimes lower) bounds for the decay rates. As a consequence of this, we show how the same initial datum can produce quantitatively and qualitatively different behavior for solutions to very similar equations.

The results described in this talk have been obtained by Lorenzo Brandolese, Maria E. Schonbek and myself through joint and individual work.