First Joint Meeting Brazil Italy of Mathematics
Special Session: Algebraic Geometry

Organized by: Marcos Jardim, Simone Marchesi and Giorgio Ottaviani

Rio de Janeiro, August 29 - September 02, 2016

Title: A duality for Calabi-Yau hypersurfaces in $\mathbb{Q}$-Fano toric varieties.
Authors: Michela Artebani
Abstract: In this talk we will present a duality between families of Calabi-Yau hypersurfaces in $\mathbb{Q}$-Fano toric varieties which generalizes the mirror symmetry construction given by Batyrev [2]. This is based on a duality between pairs (Δ_1, Δ_2) of polytopes, where Δ_1 is the Newton polytope of the family and Δ_2 is the anticanonical polytope of the ambient toric variety. Batyrev construction corresponds to the case when $\Delta_1 = \Delta_2$ is reflexive, while in case Δ_1, Δ_2 are both simplices we find a generalization of a mirror construction due to Berglund, Hübsch and Krawitz [3, 4] for certain Calabi-Yau hypersurfaces in fake weighted projective spaces. This is joint work with Paola Comparin and Robin Guilbot [1].

References

[1] M. Artebani, P. Comparin, R. Guilbot. *Families of Calabi-Yau hypersurfaces in $\mathbb{Q}$-Fano toric varieties*, to appear in J. Math. Pures Appl. 2015.

[2] V.V. Batyrev. *Dual polyhedra and mirror symmetry for Calabi-Yau hypersurfaces in toric varieties*, J. Algebraic Geom., 3, 1994, no. 3, 493–535.

[3] P. Berglund and T. Hübsch. *A generalized construction of mirror manifolds*, Nuclear Phys. B, 393, 1993, no. 1-2, 377–391.

[4] M. Krawitz. *FJRW rings and Landau-Ginzburg mirror symmetry*, ProQuest LLC, Ann Arbor, MI, 2010. Thesis (Ph.D.) University of Michigan.

Title: Limit linear series, a new perspective

Authors: Eduardo Esteves

Abstract: The notion of limit linear series was introduced by Eisenbud and Harris in the 80's to explain degenerations of linear series as smooth curves approach singular curves of compact type. Their theory has been used in several applications to explain the geometry of the moduli space of stable curves. Unfortunately though, stable curves of compact type, or more generally, treelike curves, give only divisorial information on the moduli space, thus the desire to extend the theory to all stable curves. Attempts to extend the theory have been made by several people, including the speaker, with only partial success. In this talk I will describe a new approach, that of degenerating points, to overcome the failures of past attempts. Joint work with Omid Amini (ENS, Paris).

Title: On the representation type of projective varieties

Authors: Daniele Faenzi

Abstract: In connection with the representation theory of quivers, one says that a projective variety X is of finite type if its homogeneous coordinate ring R has finitely many maximal Cohen-Macaulay (CM) indecomposable modules; also X is tame if these modules vary in families of dimension 1 at most, or wild if the dimension of these families is unbounded. I will show that, if R is CM and X is not a cone, then X is wild except for a number of completely classified cases. If time allows I will describe CM modules on a few tame varieties. [In collaboration with J. Pons-Llopis].

Title: Moduli spaces of Λ-modules on abelian varieties

Authors: Emilio Franco

Abstract: Let Λ be a D-algebra in the sense of Bernstein and Beilinson. Higgs bundles, vector bundles with flat connections, co-Higgs bundles... are examples of Λ-modules for particular choices of Λ. Simpson studied the moduli problem for the classification of Λ-modules over Kähler varieties, proving the existence of a moduli space Λ-modules. Using the Polishchuck-Rothstein transform for modules of D-algebras over abelian varieties, we obtain a description of the moduli spaces of Λ-modules of rank 1. We also proof that polystable Λ module decompose as a direct sum of rank 1 Λ-modules. This allow us to describe the module spaces of arbitrary rank in terms of a certain symmetric product. We also give a moduli interpretation of the associted Hilbert scheme. This is a joint work with Pietro Tortella.

Title: Cox rings and birational morphisms.

Authors: Antonio Laface

Abstract: Let $\mathbb{K}$ be an algebraically closed field of characteristic 0. Given a normal projective $\mathbb{K}$-variety X with finitely generated divisor class group $\mathrm{Cl}(X)$ one can define the *Cox ring* [4, 1] of X to be the graded algebra over $\mathbb{K}$:

$$\mathcal{R}(X) := \bigoplus_{[D]\in \mathrm{Cl}(X)} H^0(X, \mathcal{O}_X(D)).$$

If $\mathcal{R}(X)$ is finitely generated the variety X is a *Mori dream space*. Given a morphism $f\colon X \to Y$ of Mori dream spaces it is an open problem to relate the Cox ring of X with that of Y. In this talk, I will discuss some recent results about birational morphisms, extending those in [3, 2].

This work has been partially supported by Proyecto Fondecyt Regular N. 1150732 and Anillo ACT 1415 PIA Conicyt.

This is joint work in progress with J. Hausen and S. Keicher.

References

[1] Arzhantsev, Ivan, Derenthal, Ulrich, Hausen, Jürgen, Laface, Antonio, *Cox rings*, Cambridge Studies in Advanced Mathematics, 144, Cambridge University Press, Cambridge, 2015

[2] Arzhantsev, Ivan, Derenthal, Ulrich, Hausen, Jürgen, Laface, Antonio *Cox rings of cubic surfaces and Fano threefolds*, J. Algebra, 436, 2015, 228–276

[3] Hausen, Jürgen, Keicher, Simon, Laface, Antonio *Computing Cox rings*, Math. Comp, 85, 2016, n. 297, 467–502

[4] Hu, Yi, Keel, Sean, *Mori dream spaces and GIT*, Michigan Math J., 48, 2000, 331–348.

Title: Construction of matrices of constant rank

Authors: Paolo Lella

Abstract: A space of matrices of constant rank is a vector subspace V, say of dimension $n+1$, of the set $\mathrm{M}_{a,b}(\boldsymbol{k})$ of matrices of size $a \times b$ over a field $\boldsymbol{k}$, such that any element of $V \setminus \{0\}$ has fixed rank r. It is a classical problem, to look for such spaces of matrices, and to give relations among the possible values of the parameters a, b, r, n.

I will report on a joint work with Ada Boralevi and Daniele Faenzi, where we introduce a new effective method to construct matrices of constant rank. The starting point is to interpret the space V as an $a \times b$ matrix whose entries are linear forms and whose cokernel is a vector bundle over $\mathbb{P}^n$. Then, the main idea is that linear matrices of relatively small size can be cooked up with three ingredients, namely two finitely generated graded modules $\boldsymbol{E}$ and $\boldsymbol{G}$ over the ring $\boldsymbol{k}[x_0, \ldots, x_n]$, admitting a linear resolution up to a certain step, and a surjective map $\mu : \boldsymbol{E} \to \boldsymbol{G}$. If the sheaves $\widetilde{\boldsymbol{E}}$ and $\widetilde{\boldsymbol{G}}$ are vector bundles of rank r and s, under suitable conditions on the Betti numbers of $\boldsymbol{E}$ and $\boldsymbol{G}$ and on the map μ, the linear part of the presentation of the module $\ker \mu$ turns out to have constant rank $r - s$. Here, the module $\boldsymbol{G}$ should be thought of as a "small" modification of $\boldsymbol{E}$, as the presentation matrix of $\mathbf{G}$ is "subtracted" from that of $\mathbf{E}$.

Considering classical vector bundles over projective spaces, we produce several new examples of constant rank matrices, that are not easy to construct with previously known techniques. Finally, this technique allows to construct infinitely many examples of skew-symmetric 10×10 matrices of constant rank 8 in 4 variables; up to now, only one example of Westwick was known.

Title: Dimension counts for rational singular curves

Authors: Renato Vidal da Silva Martins

Abstract: The space $M(n,d)$ of integral projective rational curves of degree d on $\mathbb{P}^n$ can naturally be parametrized by the Grassmannian $G(n,d)$. We look for a bound for the codimension of the subvariety $M(n,d,g)$ of curves of arithmetic genus g in $M(n,d)$. When these curves are cusps, this codimension is associated to inflection (or ramification) conditions on the singularities, which are connected to the weight of such points, so that we are able to explicit an estimate. In the general case, we study the problem for small genus via stratification of the subvariety by semigroups of values. This is a joint work with Ethan Cotterill (UFF) and Lia Feital (UFV).

Title: Positivity of divisors on blown-up projective spaces

Authors: Elisa Postinghel

Abstract: The minimal model program aims at a birational classification of complex algebraic varieties. The classification of surfaces was completed in the beginning of the 20th century by the Italian school of Algebraic Geometry. In the 1980s, Mori and other researchers in this field extended the concept of minimal model to higher dimension by admitting the presence of suitable singularities. The abundance conjecture and the existence of good models are among the main open problems in the minimal model program.

In this talk we study log canonical pairs given by divisors on the blow-up of projective spaces in collections of points in general position. We give a cohomological description of the strict transforms of these divisors in the iterated blow-up along the cycles of the singular locus. Vanishing theorems for the higher cohomologies are used to give a systematic study of semi-ample divisors on these further blown-up spaces. This implies a proof of the abundance conjecture for the corresponding pairs, and an explicit construction of good minimal models.

This is a joint work with Olivia Dumitrescu.

Title: Diagonal Ideals, Isospectral Hilbert Schemes and Tautological Bundles
Author: Luca Scala
Abstract: The geometry of the big diagonal Δ_n in the product variety X^n of a smooth quasi-projective surface X is, by construction, fundamentally related to the geometry of the Hilbert scheme $X^{[n]}$ of n points over X and the isospectral Hilbert scheme B^n. We will first show how certain properties of the diagonal ideal mcI_{Δ_n}, namely the log-canonical thresholds of the pair (X^n, mcI_{Δ_n}), are related to the log-canonical thresholds of the pair $(B^n, \emptyset)$ and hence to the singularities of the isospectral Hilbert scheme B^n. We will then prove that powers of the ideal sheaf mcI_{Δ_n} correspond, under the Bridgeland-King-Reid transform, to powers of determinants of tautological bundles over the Hilbert scheme of points $X^{[n]}$; as a consequence, by comparing cohomological properties of the two sides when X is projective, we can deduce on one hand a universal formula for the Euler-Poincaré characteristic of $(\det L^{[n]})^2$ for $n \leq 4$ and on the other hand an upper bound for the Casterlnuovo-Mumford regularity of the ideal sheaves $mcI^k_{\Delta_n}$ and $(mcI^k_{\Delta_n})^{\mathfrak{S}_n}$.

Title: Geometry of the spaces of holomorphic foliations in $\mathbb{CP}^n$

Author: Israel Vainsencher

Abstract: A holomorphic foliation of codimension one and degree d in $\mathbb{CP}^n$ is defined by a 1-form $w = A_0dx_0 + ... + A_ndx_n$, where the A_i,(i=0...n), denote homogeneous polynomials of degree $d + 1$, satisfying the conditions

(i) projectivity: $A_0x_0 + ... + A_nx_n = 0$, and

(ii) Frobenius integrability: $w/\ dw = 0$.

The family of such foliations is parameterized by a closed subscheme $F(n, d)$ of a suitable $\mathbb{CP}^N$, projectivization of the space of global sections of the twisted cotangent bundle. Condition (i) (resp.(ii)) yields linear (resp.quadratic) equations for the space of foliations $F(n, d)$ in $\mathbb{CP}^N$.

The description of the irreducible components of $F(n, d)$ seems hard to tackle in full generality. For degrees $d = 0$ or $d = 1$ all components are known thanks to J. P. Jouanolou. For $d = 2$ and $n > 2$, D. Cerveau and A. Lins Neto have shown that there are just six irreducible components.

For larger degree, other components have been described by O. Calvo-Andrade, X. Gomez-Mont, A. Lins Neto, D. Cerveau, B. Edixhoven, J. V. Pereira, T. Fassarella, W. Costa e Silva, just to name a few.

Our goal is to determine the degrees of certain irreducible components of $F(n, d)$.

Title: Orthogonal and unitary tensor decomposition from an algebraic perspective
Authors: Ada Boralevi
Abstract: While every matrix admits a singular value decomposition, in which the terms are pairwise orthogonal in a strong sense, higher-order tensors typically do not admit such an orthogonal decomposition. In this talk I will present an intrinsic characterisation of those tensors that do, by means of polynomial equations of degree at most four. The exact degrees, and the corresponding polynomials, are different in each of three times two scenarios: ordinary, symmetric, or alternating tensors; and real-orthogonal versus complex-unitary. This is a joint work with J. Draisma, E. Horobeţ, and E. Robeva.