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Title: On the next-to-minimal weight of affine cartesian codes

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Abstract: Let $A_1, \ldots, A_n$ be a collection of non-empty subsets of $\mathbb{F}_q$. Consider an *affine cartesian set* $\mathcal{X} := A_1 \times \cdots \times A_n$. For a nonnegative integer d write $\mathbb{F}_q[\boldsymbol{X}]_{\leq d}$ for the $\mathbb{F}_q$-vector space formed by the polynomials in $\mathbb{F}_q[X_1, \ldots, X_n]$ of degree up to d together with the zero polynomial. We denote by d_i the cardinality of A_i, for $i = 1, \ldots, n$. Let $P_1, \ldots, P_{\widetilde{m}}$ be the points of $\mathcal{X}$. Define $\phi_d : \mathbb{F}_q[\boldsymbol{X}]_{\leq d} \to \mathbb{F}_q^{\widetilde{m}}$ as the evaluation morphism $\phi_d(g) = (g(P_1), \ldots, g(P_{\widetilde{m}}))$. The image $C_{\mathcal{X}}(d)$ of ϕ_d is a vector subspace of $\mathbb{F}_q^{\widetilde{m}}$ called the *affine cartesian code* (of order d) defined over the sets $A_1, \ldots, A_n$.

These codes were introduced in [2], and also appeared independently and in a generalized form in [1]. In the special case where $A_1 = \cdots = A_n = \mathbb{F}_q$ we have the well-known generalized Reed-Muller code of order d.

One is not only interested in the minimum distance, i.e. the minimal Hamming weight of the nonzero words of $C_{\mathcal{X}}(d)$, but also in the other Hamming weights of codewords. In this work we extend the results of Rolland in [3] to affine cartesian codes, determining the next-to-minimal weights of $C_{\mathcal{X}}(d)$ for all values of $d \geq d_1 - 1$ except in some cases where $\ell = 1$.

References

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