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Title: Using forcing to prove theorems: an example around Schanuel's conjecture
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Abstract: Schanuel's conjecture $\mathsf{SC}(\mathbb{Q}, \mathbb{C})$ is the following number theory conjecture:

Given complex numbers $a_1, \ldots, a_n$

$$\mathsf{trdg}_{\mathbb{Q}}(a_1, \ldots, a_n, \exp(a_1), \ldots, \exp(a_n)) \geq \mathsf{ldim}_{\mathbb{Q}}(a_1, \ldots, a_n).$$

Where trdg_K denotes the transcendence degree and ldim_K the linear dimension of the relevant tuple over the field K. If true, $\mathsf{SC}(\mathbb{Q}, \mathbb{C})$ would imply the algebraic independence of π over e. $\mathsf{SC}(\mathbb{Q}, \mathbb{C})$ has connections with logic as outlined by the following results (among many others):

(A) Zilber showed that there is a natural and categorical theory for algebraic closed fields of characteristic 0 with an exponential map, and this theory satisfies Schanuel's conjecture.

(B) Bays, Kirby and Wilkie showed that there is K a countable subfield of the complex numbers such that $\mathsf{SC}(K, \mathbb{C})$.

We give a new proof of **(B)** using forcing.

Shoenfield's absoluteness for Σ^1_2-statements says that if a Σ^1_2-statement can be forced, then it holds true (observe that **(B)** is a Σ^1_2-statement).

We prove that $\mathsf{SC}(\mathbb{C}^V, \mathbb{C}^{V[G]})$ holds in $V[G]$ if G is a V-generic filter for a complete boolean algebra B.

(B) will follow applying Shoenfield's absoluteness to the above result for a boolean algebra collapsing the continuum to become countable.