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Title: Modal and set theoretical tools for the study of the multiverse

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Abstract: This talk deals with logical tools devised to study different multiverses: collections of models of ZFC resulting from relative consistency proofs. The talk is divided in two parts, the first one on modal logic and the second one on set theory, that are intended to elucidate on the one hand the meta-theoretical use of consistency in independence proofs and on the other hand the notion of genericity, that is fundamental in forcing constructions.

The modal logic part deals with the modal interpretation of the \circ -operator (i.e. the consistency operator of the Logics of Formal Inconsistency) in the context of a classical negation. The resulting systems consist of modal logics that are rendered insensitive to the presence or absence of reflexivity in the accessibility relation by a suitable modification of the standard semantics. These logics were firstly introduced independently in [5] and in [3]. For this part I will present a joint work with David Gilbert (University of Illinois at Urbana-Champaign) where we show how to associate a normal modal logic L with its reflexive insensitive counterpart, which we call L° , and give a general theorem describing the conditions under which characterization results for L° follow from the analogs for L (see [1]). In the end I will hint at applications of these logics to the study of the multiverse, in the lines of [2].

The set theory part deals with the mathematical study of the generic multiverse of a countable transitive model M of ZFC. I will define, following [6], \mathbb{M}_M^Γ (i.e. the Γ generic multiverse of M) as the partial order whose elements consist of generic extension of M by means of forcings belonging to Γ and equipped with the order induced by the relation of generic extension. I will present a study the Γ generic multiverse of M , when Γ consists of the trivial and the Cohen forcing and when Γ consists of all possible forcing notions. The main result I will present deal with the possibility of building, by forcing, a truly generic model: a model built from the generic filter of \mathbb{M}_M^Γ that is not a generic extension of M .

References

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