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Title: Towards Non-deterministic algebraic semantics

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Abstract: Non-deterministic algebras (a.k.a hyperstructures, hyperalgebras, or multialgebras) are algebraic structures having at least one multivalued operation (that is, a function which assigns to any tuple of its domain a nonempty set of possible values); such multivalued operations are called hyperoperations. Multialgebras constitute an useful tool in Computer Science, apt to deal with non-deterministic processes such as non-deterministic automaton. From the perspective of Abstract Algebra, the study of hyperstructures was initiated by F. Marty in 1934, when he proposed the notion of hypergroups. Afterwards, several algebraic hyperstructures have been studied in the literature besides hypergroups: hyperlattices, hyperrings, hyperfields, and hyperalgebras in general.

A Non-deterministic matrix, or Nmatrix, is a multialgebra together with a set of designated values of its domain. Nmatrices constitute the natural application of multialgebras to Logic. They were introduced in 1962 by N. Rescher under the name of quasi-truth-functional systems. This notion was independently rediscovered by J. Kearns in 1981 and by Y. Ivlev in 1988, as a way to overcome Dugundji's results on modal logic by means of an alternative to Kripke semantics. The Nmatrix semantics was reintroduced once again (together with the terminology "non-deterministic matrices" and "Nmatrices") by A. Avron and I. Lev in 2001, and afterwards intensively developed, from the point of view of applications, by A. Avron and his collaborators. In particular, they shown that several paraconsistent logics in the hierarchy known as Logics of Formal Inconsistency (LFIs), which cannot be characterized by a single finite matrix (that is, systems under the scope of a Dugundji-like theorem), can be characterized by a single finite Nmatrix. Avron's results were recently generalized by W. Carnielli and M. Coniglio by means of the concept of swap structures, which are multiagebras defined over 3-fold products of Boolean algebras. This structures are related, and generalize in some sense, the well-known Fidel-Vakarelov twist structures.

In this talk, new completeness results by means of swap structures will be presented for some LFIs and non-normal modal logics, by introducing the notion of Lindenbaum-Tarski multialgebras. The techniques introduced here open the possibilities of defining a new

notion of algebraizability of logics based on classes of multialgebras. This method could be appliable to logics which cannot be agebraizable by means of traditional tools, including the very general theory of Blok-Pigozzi.

This is a joint work with Ana Claudia Golzio.