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Title: One more step

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Abstract: We know from Simpson's monograph, and, perhaps, from elsewhere, that ATR_0 allows to produce a good theory of V -countable ordinals, transitive sets and imitate the cumulative hierarchy of sets well (resembling apparent V -hereditarily countable sets).

In this set-up, appropriately encoded, we can develop the theory of rudimentary functions, the definability predicate, Gödel's L and elements of the fine structure theory. It is also possible in some very special models of second-order arithmetic, while constructing their L , that formerly V -countable ordinals lose their countability and become L -uncountable cardinals. (We get ZFC minus Powerset.) (Imagine that the bijection with omega stayed outside L .)

I am planning to show a few new tricks (by adding some finite amount of L -indiscernibility with small parameters), to make these L -cardinals into large cardinals below $0\#$, and thus produce a factory of rough versions of Harvey Friedman's most difficult and technically impressive discovery — the unprovability proofs ("reversals") at the levels of n -SRP and $0\#$.

The power-set axiom, the L -indiscernibility and the appropriate large cardinal properties will follow from a certain second-order arithmetical "homogeneity" principle, like in Ramsey theory.

All previous existing proofs of these "reversals" by Harvey Friedman are longer than 100 pages, so my new proof-elements or tricks provide a shortening of the best metamathematical arguments of nowadays. We pay for that by apparent ugliness of our (ZFC + large cardinals) – unprovable statements and, usually, by absence of Friedman's universality (fixed large dimension n of n -SRP).

20% of this study comes from learning the Manchester ways of doing model theory, another 40% comes from studying Harvey Friedman's manuscripts available on his homepage, 10% comes from studying Devlin's book, the rest is original.

If time allows, I will also mention the recent developments on n -baby measurable cardinals, a joint research with Zachiri McKenzie. These cardinals (between Victoria Gitman's super-Ramsey cardinals and measurable cardinals) have neat combinatorial and embed-

ding properties and characterise the strengths of NFUM and KMU and their fragments, answering open questions of R. Holmes from 1990. These cardinals are the next candidates after n -Mahlo and n -SRP to be subjected to Friedman-style arithmetical model-theoretic analysis (this time not compatible with $V = L$).

It became common among metamathematicians to think that the essence of all respectable unprovability proofs nowadays comes from the mantra dating to Paris-Harrington style (and subsequent) proofs, most of them forgotten, (“the witness of induction (least element), collection, replacement or comprehension always comes before the next indiscernible”).

I, in this talk, tentatively agreeing with this philosophy as much as current practice allows me to, will concentrate on other aspects of unprovability. However this mantra has to be mentioned as something I am not going to discuss.

The talk will demand as pre-requisites, the knowledge of some modern metamathematics (first-order and second-order), a bit of model theory of PA and ZFC, and the knowledge of the definition of L .

I shall not talk about the crucial notions of Ignorabimus or Arithmetical Splitting this time, although they, of course, are the main motivations and main goals of logic and metamathematics in the next few decades, and the main motivations of my own research.