

First Joint Meeting Brazil–Italy in Mathematics  
Special Session: GEOMETRIC STRUCTURES, LIE THEORY  
AND APPLICATIONS

Rio de Janeiro, August 29 - September 02, 2016

**Title:** Integrals on Grassmannians of Lines

**Authors:** Letterio Gatto (Politecnico di Torino)

**Abstract:** Let  $G(r, n)$  be the complex Grassmann variety parametrizing  $r$ -dimensional subspaces of  $\mathbb{C}^n$ . By an integral on  $G(r, n)$  one usually means the degree of a product of Chern classes of the universal quotient bundles  $\mathcal{Q}$  over it. The most popular is  $\int_{G(r, n)} c_1(\mathcal{Q})^{r(n-r)} \cap [G(r, n)]$ , which coincides with the Plücker degree of  $G(r, n)$ . Within the framework of Gaudin models and representation theory of the Lie algebra  $\mathfrak{sl}_2(\mathbb{C})$ , basing on previous work with Varchenko [5] about critical points of the generating function of the Wronski map, I. Scherbak proves in [4] a formula computing arbitrary integrals on  $G(2, n+2)$ . A purely algebraic proof of Scherbak’s result will be offered in the talk by exploiting the fact that the Chern polynomial  $c_t(\mathcal{Q})$  defines a Hasse-Schmidt derivation on a Grassmann algebra, in the sense of [1]. In addition, if time permits, we shall briefly discuss i) the fact, observed by Santiago, that the generating function of the degrees of the Grassmannian  $G(2, n)$  of lines in  $\mathbb{P}^{n-1}$  can be expressed in terms of modified Bessel functions of the first kind and ii) the relationship of the subject with the combinatorics of the so called Catalan traffic [2, 3].

## References

- [1] L. Gatto, P. Salehyan, *Hasse-Schmidt Derivations on Grassmann Algebras*, Impa Monographs n. 4, Springer, 2016
- [2] H. Niederhausen, *Catalan Traffic at the Beach*, The Eletronical Journal of Combinatorics, **9**, (R33) (2002), 1–17.
- [3] T. Santiago, “*Catalan traffic*” and integrals on the Grassmannian of lines, *Discrete Mathematics* **308** (2008) 148–152.
- [4] I. Scherbak, *Gaudin Models and the generating function of the Wronski map*, *Geometry and topology of Caustics*, 2002, Banach Center Publications, Vol **62**, 2002, 249–262.
- [5] I. Scherbak, A. Varchenko, *Critical points of functions,  $\mathfrak{sl}_2$  representations, and Fuchsian differential equations with only univalued solutions*, *Mosc. Math. J.* **3** (2003), no. **2**, 621–645.