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**Title:** A zoo of symplectic, complex and non-Kähler manifolds

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**Abstract:** It is well known that a Kähler manifold has an underlying symplectic structure. Forty years ago Thurston provided the first example of a compact symplectic manifold that admits no Kähler metrics. Giving examples of these manifolds is hampered by a twofold problem. On the one hand, constructing compact symplectic manifolds is a challenging task: among the available techniques, we mention the symplectic blow-up of Gromov and McDuff, the symplectic connected sum of Gompf and the asymptotically holomorphic theory of Donaldson. A slightly more difficult problem is to construct manifolds which, although not Kähler, are at the same time symplectic *and* complex. On the other hand, one needs to know whether the existence of a Kähler structure influences the underlying smooth manifold, and how. The Hard Lefschetz property and the formality of the rational homotopy type answer this second problem.

The talk surveys examples of manifolds which are complex and symplectic and do not carry a Kähler metric, emphasizing the role played by the fundamental group. The only example missing in this bestiary is that of a six-dimensional simply connected, compact, symplectic and complex manifold which admits no Kähler metrics. Based on the preprint [BFM], we complete the picture by constructing such missing exemplar.

[BFM] G.Bazzoni, M.Fernández and V.Muñoz, *A 6-dimensional simply connected complex and symplectic manifold with no Kähler metric*, preprint <http://arxiv.org/abs/1410.6045>