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Title: Gelfand–Kirillov dimension of relatively free Lie algebras

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Abstract: Let K be a field and let $F(M_n(K))$ be the relatively free algebra in the variety generated by the matrix algebra $M_n(K)$. Procesi developed a powerful theory for studying $F(M_n(K))$ which allowed employing methods from Commutative algebra to be applied to the study of rings with polynomial identities. As a consequence Procesi computed the Gelfand–Kirillov dimension of $F(M_n(K))$ (in characteristic 0). Later on Berele showed that one can modify Procesi's proof so that it works when K is an infinite field. Berele proved that the GK dimension of a finitely generated PI algebra is always finite. A theorem of Belov states that the GK dimension of a relatively free (finitely generated) relatively free algebra is always an integer. The GK dimensions of the relatively free algebras of T-prime algebras are known.

While in the associative case a lot of information concerning GK dimension of associative algebras has been gathered very little is known in the case of Lie algebras. Around 1970, Bahturin computed the GK dimension of the relatively free algebra in two generators for $sl_2(K)$ in characteristic 0. In this talk we compute the GK dimension of $F(sl_2(K))$ in the general case. The methods are based on the theory developed by Procesi, and thus we hope we would be able to extend the result to $sl_n(K)$. Our proof works whenever K is an infinite field of characteristic different from 2.