On exponent matrices of tiled orders

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in collaboration with

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A ring $\Lambda$ is called a tiled order if $\Lambda$ is a prime Noetherian semi-prefect semi-distributive ring with non-zero Jacobson radical. Any tiled order can be constructed from a (non-necessarily commutative) discrete valuation ring and an exponent matrix. The latter means a square integer matrix $A = (\alpha_{p_{ik}})$, whose diagonal entries are equal to zero, and for all possible indices $i$, $j$, $k$, the following inequality holds:

$$\alpha_{ij} + \alpha_{jk} \geq \alpha_{ik}.$$  

Every tiled order $\Lambda$ is isomorphic to a ring of the form

$$\Lambda = \sum_{i,j=1}^{n} e_{ij}(\pi^{\alpha_{ij}} O) \subseteq M_n(O),$$

where $n \geq 1$, $O$ is a (non-necessarily commutative) discrete valuation ring with prime element $\pi$, $(\alpha_{ij})$ is an exponent matrix, $e_{ij}(\pi^{\alpha_{ij}} O) = \{e_{ij}(a), a \in \pi^{\alpha_{ij}} O\}$ and $e_{ij}(a)$ is the $n \times n$-matrix whose unique non-zero entry $a$ is placed in the $(i, j)$-position.


We shall present some results in collaboration with G. Kudryavtseva, V. Kirichenko and M. Plakhotnyk. In particular we endow the set of non-negative exponents matrices $E_n$ with the following two operations: the component-wise addition, which we denote by $\circ$ and the component-wise maximum, which we denote by $\oplus$. It follows that $(E_n, \circ, \oplus, 0)$ is a max-plus algebra of matrices where $0$ denotes the zero matrix. Most of usual axioms of an idempotent semiring hold in this algebra: both operations are associative, commutative, $0$ is the neutral element with respect to each of these operations, $\oplus$ is idempotent and $\circ$ distributes over $\oplus$.

One of our results provides a basis for this max-plus algebra.

References

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