On exponent matrices of tiled orders

M. Dokuchaev
in collaboration with
G. Kudryavtseva, V. Kirichenko and M. Plakhotnyk.

A ring Λ is called a *tiled order* if Λ is a prime Noetherian semi-prefect semi-distributive ring with non-zero Jacobson radical. Any tiled order can be constructed from a (non-necessarily commutative) discrete valuation ring and an exponent matrix. The latter means a square integer matrix $A = (\alpha_{ps})$, whose diagonal entries are equal to zero, and for all possible indices i, j, k, the following inequality holds:

$$\alpha_{ij} + \alpha_{jk} \geq \alpha_{ik}.$$

Every tiled order Λ is isomorphic to a ring of the form

$$\Lambda = \sum_{i,j=1}^{n} e_{ij}(\pi^{\alpha_{ij}}\mathcal{O}) \subseteq M_n(\mathcal{O}), \tag{1}$$

where $n \geq 1$, $\mathcal{O}$ is a (non-necessarily commutative) discrete valuation ring with prime element π, (α_{ij}) is an exponent matrix, $e_{ij}(\pi^{\alpha_{ij}}\mathcal{O}) = \{e_{ij}(a), a \in \pi^{\alpha_{ij}}\mathcal{O}\}$ and $e_{ij}(a)$ is the $n \times n$-matrix whose unique non-zero entry a is placed in the (i, j)-position.

Since the paper by R. B. Tarsy [8] on global dimension of orders published in 1970, tiled orders draw attention of a number of experts in ring theory and representation theory (see, in particular, [1] – [7]).

We shall present some results in collaboration with G. Kudryavtseva, V. Kirichenko and M. Plakhotnyk. In particular we endow the set of non-negative exponents matrices $\mathcal{E}_n$ with the following two operations: the component-wise addition, which we denote by $\odot$ and the component-wise maximum, which we denote by $\oplus$. It follows that $(\mathcal{E}_n, \odot, \oplus, 0)$ is a max-plus algebra of matrices where 0 denotes the zero matrix. Most of usual axioms of an idempotent semiring hold in this algebra: both operations are associative, commutative, 0 is the neutral element with respect to each of these operations, $\oplus$ is idempotent and $\odot$ distributes over $\oplus$.

One of our results provides a basis for this max-plus algebra.

References

[1] Dokuchaev, M., Kirichenko, V., Polcino Milies, C., Locally nilpotent groups of units in tiled rings, *J. Algebra* **323**, (2010), no. 11, 3055-3066.
[2] H. Fujita, Tiled orders of finite global dimension, *Trans. Amer. Math. Soc.* **322** (1990) 329–342.
[3] H. Fujita, Neat idempotents and tiled orders having large global dimension, *J. Algebra* **256** No 1 (2002) 194–210.
[4] W. S. Jansen and C. J. Odenthal, A tiled order having large global dimension, *J. Algebra* **192** No. 2 (1997) 572–591.
[5] V. A. Jategaonkar, Global dimension of tiled orders over a discrete valuation ring, *Trans. Amer. Math. Soc.* **196** (1974) 313–330.
[6] E. Kirkman and J. Kuzmanovich, Global dimensions of a class of tiled orders, *J. Algebra* **192** no. 2 (1997) 572–591.
[7] V. V. Kirichenko, M. A. Khibina, K. W. Roggenkamp and V. N. Zhuravlev, Gorenstein tiled orders, *Commun. Algebra* **29** No. 9 (2001) 4231–4247.
[8] R.B. Tarsy, Global dimension of orders, *Trans. Amer. Math.Soc.* **151** (1970) 335–340.

Departamento de Matemática, Universidade de São Paulo, Brazil