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Title: Ergodic theory, thermodynamic formalism and transport theory applied to some cooperative and non-cooperative games

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Abstract: Let $T : X \to X$ and $S : Y \to Y$ be continuous maps defined on compact sets. Let

$$\varphi_i(\mu, \nu) = \int_{X \times Y} A_i(x, y) d\mu(x) d\nu(y) \text{ for } i = 1, 2,$$

where μ is a T-invariant measure and ν is a S-invariant measure, be *pay-off functions* for a game (in the usual sense of game theory) between players that have the set of probability invariant measures for T (player 1) and S (player 2) as possible *strategies*. Our goal here is to establish the notion of Nash equilibrium point for the game defined by this pay-offs and strategies. The main tools came from ergodic optimization (as we are optimizing over the set of invariant measures) or thermodynamic formalism (when we add to the integrals above the entropy of measures in order to define a second case to be explored). Both cases are ergodic versions of non-cooperative games. We show the existence of Nash equilibrium points with two independent arguments. We also present examples and briefly discuss uniqueness (or lack of uniqueness). In the end we present a different example where players are allowed to collaborate. This final example show connections between cooperative games and ergodic transport, a recent area of research where one wants to transport, at minimal cost, a fixed measure into a measure which is invariant for some dynamical system.