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Title: Mean Field Games with state constraints

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Abstract: We are interested in MFG problems with state constraints in $\overline{\Omega}$, where Ω is an open bounded subset of $\mathbb{R}^d$. Heuristically this game with infinitely many players runs as follows. The repartition of the player at initial time $t = 0$ is m_0 where $m_0 \in \mathcal{P}(\overline{\Omega})$, the space of probability measures on $\overline{\Omega}$. Let $t \to m(t)$ be a measurable trajectory from $[0, T]$ to $\mathcal{P}(\overline{\Omega})$. Any player starting from $x \in \overline{\Omega}$ solves an optimal control problem of the form

$$u(0, x) := \inf_{\gamma,\ \gamma(0)=x} \int_0^T L(\gamma(t), \dot{\gamma}(t)) + F(\gamma(t), m(t))\ dt + G(\gamma(T), m(T)),$$

where the infimum is restricted to the state constraint $\gamma(t) \in \overline{\Omega}$ for all t. Let $\overline{\gamma}^x$ be an optimal solution starting from x. An equilibrium configuration should satisfy $m(t) = \overline{\gamma}^{\cdot}(t)\sharp m_0$ for all $t \in [0, T]$. Unlike the unconstrained case, the problem with this formulation is that there might be several optimal solutions to the optimal control problem for a large set of initial conditions. We will show how to relax the problem to prove the existence of an equilibrium. Then we'll discuss the uniqueness and regularity issues.