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Title: Morse homology and problems of prescribed mean curvature.

Author: Graham Smith

Abstract: The construction of constant curvature hypersurfaces subject to geometric or topological restrictions is a standard one of riemannian geometry. Closely related to this is the problem of constructing hypersurfaces of curvature *prescribed* by some function of the ambient space. In fact, a complete understanding of the one generally entails a complete understanding of the other.

In constructing constant and prescribed curvature hypersurfaces, the techniques of infinite-dimensional differential topology have yielded many pleasing existence results (c.f. the work of Jost, Schneider, Tomi, Tromba, White, and so on). However, in various interesting cases, the number of solutions that these techniques promise is (algebraically!) equal to zero. This is clearly not very helpful. To get round this, we aim to develop a deeper, Morse homology theory, built around forced mean curvature flows, which, by counting separately solutions of different Morse index, should yield a far greater number of solutions.

In the present exposé, we show the effectiveness of this technique in the case where the mean curvature is prescribed by a suitably controlled, positive function, and the ambient space is a flat $(d+1)$-dimensional torus. In this manner we show that, for generic data, there exist at least 2^{d+1} locally strictly convex, immersed hyperspheres with mean curvature prescribed by that data.