

First Joint Meeting Brazil Italy of Mathematics Special Session: Population Dynamics and Evolution

Rio de Janeiro, August 29 - September 02, 2016

Title: Evolution of cooperation in a particular case of the infinitely repeated Prisoner's Dilemma with three strategies

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Abstract: We study a population of individuals playing the infinitely repeated Prisoner's Dilemma under replicator dynamics. The population consists of three kinds of individuals adopting the following reactive strategies: ALLD (individuals which always defect), ATFT (almost tit-for-tat: individuals which almost always repeat the opponent's last move) and G (generous individuals, which always cooperate when the opponent cooperated in the last move and have a positive probability q of cooperating when their opponent has defected). Our aim is studying in a mathematically rigorous fashion the dynamics of a simplified version for the computer experiment in [Nowak MA, Sigmund K (1992) *Tit for tat in heterogeneous populations*, Nature 355:255–253] involving 100 reactive strategies. We see that as the generosity degree q of the G individuals varies, equilibria (rest points) of the dynamics appear or disappear, and the dynamics changes accordingly. We prove that the results of the experiment are true in our simplified version: cooperation may evolve if q is smaller than a threshold. But we also have complete control on the existence or non-existence of the equilibria for the dynamics for all possible values of the parameters, given that ATFT individuals are close enough to TFT. For most values of the parameters the dynamics can be completely determined.