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Title: Orderable groups and Additive number theory

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Abstract: Let G denote an arbitrary group.

If S is a subset of G , we define its square S^2 by $S^2 = \{x_1x_2 \mid x_1, x_2 \in S\}$.

We are concerned with the following general problem:

let S be a finite subset with k elements of a group G , determine the structure of S , if $|S^2|$ satisfies the following inequality: $|S^2| \leq \alpha|S| + \beta$ for some small $\alpha \geq 1$ and small $|\beta|$.

Problems of this kind are called *inverse problems of small doubling type*. They have been first investigated by G.A. Freiman in the additive group of the integers; many authors are now extending the classical Freiman's inverse theorems (see [1]) to nonabelian groups.

In this talk we will survey some recent results, contained in [2]-[7], concerning the structure of finite subsets S of an orderable group satisfying the small doubling property with $\alpha = 3$ and small $|\beta|$'s.

References:

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[4] G.A. Freiman, M. Herzog, P. L., M. M., Y.V. Stanchescu, *A small doubling structure theorem in a Baumslag-Solitar group*, European J. Combin., **44** (2015), 106-124.

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[6] G.A. Freiman, M. Herzog, P. L., M. M., A. Plagne, Y.V. Stanchescu, *Small doubling in ordered groups: generators and structures*, submitted.

[7] G.A. Freiman, M. Herzog, P. L., M. M., Y.V. Stanchescu, *Small doubling in ordered nilpotent group of class 2*, submitted.