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Seifert matrices of braids with applications to isotopy and signatures

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Abstract. Let β be a braid with closure $\widehat{\beta}$ a link. Collins developed an algorithm to find the Seifert matrix of the canonical Seifert surface Σ of $\widehat{\beta}$ constructed by Seifert's algorithm. Motivated by Collins' algorithm and a construction of Ghys, we define a 1-dimensional simplicial complex $K(\beta)$ and a bilinear form $\lambda_\beta : C_1(K(\beta);\mathbb{Z}) \times C_1(K(\beta);\mathbb{Z}) \to \mathbb{Z}[\frac{1}{2}]$ such that there is an inclusion $K(\beta) \hookrightarrow \Sigma$ which is a homotopy equivalence inducing an isomorphism $H_1(\Sigma;\mathbb{Z}) \cong H_1(K(\beta);\mathbb{Z})$ such that $[\lambda_\beta] : H_1(K(\beta);\mathbb{Z}) \times H_1(K(\beta);\mathbb{Z}) \to \mathbb{Z} \subset \mathbb{Z}[\frac{1}{2}]$ is the Seifert form of Σ. We show that this chain level model is additive under the concatenation of braids and then verify that this model is chain equivalent to Banchoff's combinatorial model for the linking number of two space polygons and Ranicki's surgery theoretic model for a chain level Seifert pairing. We then define the chain level Seifert pair (λ_β, d_β) of a braid β and equivalence relations, called A and $\widehat{A}$-equivalence. Two n-strand braids are isotopic if and only if their chain level Seifert pairs are A-equivalent and this yields a universal representation of the braid group. Two n-strand braids have isotopic link closures in the solid torus $D^2 \times S^1$ if and only if their chain level Seifert pairs are $\widehat{A}$-equivalent and this yields a representation of the braid group modulo conjugacy. We use the first representation to express the ω-signature of a braid β in terms of the chain level Seifert pair (λ_β, d_β).

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