



# Isometric embeddings of the square flat torus in ambient space

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**Abstract.** This memoir is concerned with isometric embeddings of a square flat torus in the three dimensional Euclidean space. The existence of such embeddings was proved by John Nash and Nicolaas Kuiper in the mid 50s. However, the geometry of these embeddings could barely be conceived from their original papers. Here we provide an explicit construction based on the convex integration theory introduced by Mikhail Gromov in the 70s. We then turn this construction into a computer implementation leading us to the visualisation of an isometrically embedded flat torus. The pictures reveal a geometric object in-between fractals and ordinary surfaces. We call this object a  $C^1$  fractal.