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Some properties of the Cremona group

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Abstract. We recall some properties, unfortunately not all, of the Cremona group.

We first begin by presenting a nice proof of the amalgamated product structure of the well-known subgroup of the Cremona group made up of the polynomial automorphisms of $\mathbb{C}^2$. Then we deal with the classification of birational maps and some applications (Tits alternative, non-simplicity...) Since any birational map can be written as a composition of quadratic birational maps up to an automorphism of the complex projective plane, we spend time on these special maps. Some questions of group theory are evoked: the classification of the finite subgroups of the Cremona group and related problems, the description of the automorphisms of the Cremona group and the representations of some lattices in the Cremona group. The description of the centralizers of discrete dynamical systems is an important problem in real and complex dynamic, we describe the state of the art for this problem in the Cremona group.

Let S be a compact complex surface which carries an automorphism f of positive topological entropy. Either the Kodaira dimension of S is zero and f is conjugate to an automorphism on the unique minimal model of S which is either a torus, or a K3 surface, or an Enriques surface, or S is a non-minimal rational surface and f is conjugate to a birational map of the complex projective plane. We deal with results obtained in this last case: construction of such automorphisms, dynamical properties (rotation domains...).

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